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**SIGNIFICANCE OF VOLATILITY COMPONENTS IN PRICING: CASE FOR AN
EMERGING MARKET**

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

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04/07/2016

This Research Proposal has been submitted for examination with my approval as the Supervisor.

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Abstract

This research is an investigation of the volatility structure of the Kenyan financial market and its influence on stock returns. Using the little-known component GARCH, I decompose market volatility into its long run and short run components then examine their significance in explaining stock returns in a multifactor model.

Performance of this model is also compared against the decades old Sharpe and Linter (1965) CAPM. Issues of whether the component GARCH accurately captures the volatility structure of the Kenyan Stock Exchange are highlighted. The results also allow some inference into the interaction between the business cycle and market movements of a developing country like Kenya. Results have been made robust by implementing the model using both daily and weekly data.

keywords: component GARCH, volatility structure, financial constraints

JEL classification: G12; G1

1 INTRODUCTION

1.1 Background

Investors are concerned primarily with the risk that their investments bear. The securities market instruments are no exception to this concern. Careful determination of risk in any stock is therefore paramount. This is ever more important if quantifying the risk can lead to estimation of an expected return. With a good estimation of the return, an “appropriate” or more reflective price of the asset can be derived by an investor. This is the essence of asset pricing theory; understanding the values of claims to uncertain payments (Cochrane, 2001).

Measuring of risk in an asset is conventionally done through its volatility, which is the standard deviation. Consequently, most pricing models will include this measure in some form or account for the sources of that volatility. These models majorly focus on pricing exchange traded instruments because of the large data set available on them. Pricing models can be described as absolute or relative. Relative models determine the price an asset based on the relative prices of other assets whereas absolute models look at the return driving factors of an asset to determine the price. Absolute pricing can be done through the notable multifactor models.

Multifactor models derive expected stock returns from a set of return drivers which are assumed to fully explain the fluctuations in asset returns. They are categorized into: macroeconomic, statistical and fundamental factor models. The first type uses observable economic variables as return driving factors for stocks. Statistical models use the asset returns with the tools of maximum likelihood and principal components to maximize explanatory power. Fundamental factor models however, rely on company attributes such as firm size, dividend yield etc. to determine the expected return.

Comparative performance of the three models is an important aspect to be considered. Due to the many problems of economic data (for instance errors in measurement, frequent restating etc.) which are not present or rather not so severe in fundamental or statistical factors; as a corollary then macroeconomic factor models should be out performed by the rest. In fact, empirical evidence supports this (Connor, 1995).

Significant work has been done on all three forms of models with a lot of interest going especially to statistical models because of the many different approaches that can be taken.

Recently it has been shown by (Adrian & Rosenberg, 2008) that a multifactor model that includes short term and long run volatility together with the market return can outperform the famous CAPM¹ model

The CAPM is an elegant single factor model developed by (Sharpe, 1964), (Lintner, 1965) and a modified version by (Black, 1972). Because it was derived from the widely accepted mean variance framework of Harry Markowitz it was praised and was expected to perform well in explaining returns. The reality however was not so rosy. Most of the implications that the CAPM model predicted were tested and the results were not in full support of CAPM (Fama & French, 2004).

From the tenets of asset pricing theory, the demand for assets in the current period is determined by the expected consumption in future periods. Thus assets that covary more with consumption have less demand than those that covary less; this leads to lower prices. Similarly, they will have higher risk as shown in (Cochrane, 2001) and (The Royal Swedish Academy of Sciences, 2013). Indeed indicators of this covariation should then be used in any pricing kernel.

Volatility of the stock market has been shown to be affected by the business cycle which portrays the level of economic activity hence consumption in the economy (Campbell, Lettau, Malkiel, & Xu, 2001). Naturally, volatility of the market should then be included in the pricing kernel.

1.2 Problem Statement and Justification

Factors that affect the systematic risk of an asset will also affect its return (Merton, 1973). Market volatility has been shown by (Ang, Hodrick, Zhang, & Xing, 2006) to be a significant factor in explaining returns. The relationship has been further supported by (Adrian & Rosenberg, 2008), who decompose market volatility² into its short term and long term components. They also show that a factor model using both components³ of volatility and the market return does in fact empirically outperform the CAPM.

¹ CAPM stands for Capital Asset Pricing Model

² Henceforth referred to as “volatility”

³ To prevent repetitiveness, the word “components” shall be used henceforth to refer to long run and short run volatility components.

However, their research and conclusions are subject to the markets that they analyzed and the methods they used. They based their research in developed markets which differ greatly with emerging markets in income level and technology just to mention a few. Emerging markets have higher financial constraints, lower income per capita, limited financial instruments, lower participation in the financial markets etc. Attributes of volatility in emerging markets also differ from those of developed markets (Morck, Yeung, & Yu, 2013). All these features will in turn affect the behavior of prices in stock markets.

It follows that the conclusions reached by the previous authors might differ significantly when the analysis is performed in emerging markets. If so, then Kenya being an emerging market might also show altered results. This project is an attempt to test that hypothesis by decomposing the volatility components of the Kenyan NSE⁴ and comparing their explanatory power against that of the CAPM.

1.3 Research questions

1. Is there a linear relationship between volatility components and stock returns?
2. Does a model using APT⁵ framework that includes the volatility components perform better than the CAPM in explaining stocks in the NSE?

1.4 Research objectives

1. To determine the significance of volatility components in explaining stock returns.
2. To compare the performance of the CAPM versus the volatility component model in explaining returns.

⁴ Nairobi Securities Exchange

⁵ Arbitrage pricing Theory, see (Ross, 1976)

2 LITERATURE REVIEW

2.1 Introduction

In the following section, previous literature in asset pricing, multifactor models and market volatility modelling is reviewed. The implication of the results from the literature are highlighted and constraints of their methodology to similar work on the Kenyan NSE data are elicited.

2.2 CAPM Rise and Fall

After Harry Markowitz's work (Markowitz, 1952) that showed diversification could lead to elimination of risk in a portfolio there remained a gap on how the remaining systematic risk after diversification could be related to the price of an asset in equilibrium. This gap was filled by a theory developed by (Sharpe, 1964) and (Lintner, 1965) with their famous Capital Asset Pricing Model. This model gave a measure for systematic risk as the covariance between the market portfolio's⁶ return and the assets return divided by the market portfolio's variance. Using a simple linear relationship the return premium on an assets could be calculated given the market portfolio's return. This market portfolio was assumed to be efficient but cannot be tested outright for efficiency.

By adding a few more assumptions to the Markowitz framework which are:

1. Investors are in complete agreement about the distribution of returns and this distribution is the true one and,
2. Borrowing and lending occurs at the risk free rate,

The CAPM equation can be derived. If risk free borrowing and lending is not available then the Black CAPM with a zero beta portfolio is used instead. In the seminal paper (Fama & French, 2004), they survey most of the literature available on tests of the CAPM and show that its performance is not satisfactory. Tests have relied on three implications of the CAPM:

1. A linear relationship between asset prices and their betas and there's no other variable with marginal explanatory power.
2. A positive equity premium⁷.
3. Assets that are not correlated with the market return earn the risk free rate of return.

⁶ An efficient Portfolio of all risky assets.

⁷ The market portfolio excess return over the risk free rate.

Both cross sectional and time series tests only have weak support for the CAPM. For instance, cross sectional regressions show that the average intercept is higher than the risk free rate and the beta coefficient is lower than the average equity premium. For the (Sharpe, 1964) and (Lintner, 1965) version, most tests reject that the market portfolio has enough explanatory power (Fama & French, 2004).

2.3 Other Factors Considered

From that last result, it follows that by adding more variables to the Sharpe Lintner⁸ version would increase the explanatory power. But this is feasible only if the right variables are chosen. This proposition was further supported by the arbitrage pricing theory of (Ross, 1976), that long term asset excess returns over the risk free rate could be related to a set of factors whether statistical, fundamental or macroeconomic.

Many factors have been considered by researchers and their relative performance compared. From interest rates to inflation to market return skewness; researchers leave no stone unturned. The search for the most practical pricing factors for assets has not abated in zeal nor tapered in scope. As a consequence, several models have been developed. For instance, the Chen Roll Ross model (Chen, Ross, & Roll, 1986) that suggested factors that affect the discount rate⁹ and expected cash flows to be those that affect the return as explanatory factors; specifically these factors were industrial production, inflation, bond premia, oil prices and market indices.

Even with this seminal work, further research to find more suitable factors was done. Most notable is the Fama French five factor model (Fama & French, 1993) which provides empirical argument for firm size, book to market equity, the market portfolio, bond maturity and default risks as possible factors. This was an extension of their previous three factor model (Fama & French, 1992).

2.4 Volatility: Attributes and Explanatory power

As more factors were considered, more attention was paid to statistical factors. This led to the great contribution of (Ang, Hodrick, Zhang, & Xing, 2006) that included volatility as an explanatory factor.

⁸ This refers to (Sharpe, 1964) and (Lintner, 1965)

⁹ The rate that can be used to determine present values of future cash flows.

Aggregate volatility¹⁰ is a significant pricing factor in the determination of stock returns as shown by (Ang, Hodrick, Zhang, & Xing, 2006). In their paper, they set out to determine whether volatility is a priced factor in cross section of expected returns and find out the price of aggregate volatility. Their results conclude that stocks with high sensitivities to innovations (shocks) in aggregate volatility have low average returns. They also find a negative relation between idiosyncratic risk and average stock returns which is contrary to results from other researchers' work most notable being (Lintner, 1965).

Their methodology does not model volatility evolution using the famous ARCH models but instead uses direct observations for the VIX Index which captures implied volatility from stock options. This however, forced them to limit their sample size (because the VIX series does not extend to periods before option trading) and as a result they speculate in their conclusion that the negative price of risk found could have changed sign if a larger sample was used to include more periods of shock. For markets without a volatility index replicating this research would not be possible. Their model of market return¹¹ and market volatility to explain returns achieved lower pricing errors than the CAPM.

Given these results of (Ang, Hodrick, Zhang, & Xing, 2006); (Adrian & Rosenberg, 2008) have furthered the work of using volatility to explain stock returns by decomposing the volatility into its long run and short term components and added them to the market portfolio to explain returns model. The result is lower pricing errors than the CAPM and the Ang et al (2006) model.

The authors show presence of variation in the sensitivities of various portfolios to both components. The average sensitivity of both components is negative, suggesting that higher volatility leads to lower expected return. This is contrary to what one might expect that higher volatility meaning higher risk should lead to higher expected return but it is however consistent with the results of their predecessors (Ang, Hodrick, Zhang, & Xing, 2006).

The observed relationship is tied to economic thinking by using short run component as a measure of financial constraints and the long run component as a measure of the business cycle. The latter relation is in agreement with the observations of (Campbell, Lettau, Malkiel, & Xu, 2001) discussed later.

¹⁰ This is the overall market volatility.

¹¹ The market portfolio

Greater financial constraints in emerging markets leads to the expectation that the analysis of this project to conclude that the average short run volatility risk premium to be higher than the 0.17% observed by (Adrian & Rosenberg, 2008). The methodology used may however differ.

Asymmetric volatility is the phenomenon in equity securities that negative shocks lead to more volatility than positive ones. It is observed in both mature and emerging markets although to different extents as shown by (Jayasuriya, Shambora, & Rossiter, 2009). Possible explanations of the phenomenon have included the leverage effect and volatility feedback.

For leverage effect, falls in share prices lead to increase in the leverage ratio, thus increasing the stock's risk. This makes stocks riskier and as a result volatility increases. The problem with this is that daily stock price changes are too low to impact the leverage ratio significantly thus it cannot account for the increase in volatility. Volatility feedback on the other hand is due to the risk averse nature of most investors. With a convex indifference curve of return and volatility, an anticipated increase in volatility raises the required return on equity, thus lowering the price which is a negative shock that leads to further increases volatility.

The authors (Jayasuriya, Shambora, & Rossiter, 2009) measure the magnitude of asymmetric volatility and determine its trend in 14 emerging markets and 7 mature ones in three equal sub periods from 1992 to 2007. They use the power GARCH model parameters as a measure of asymmetric volatility. The results indicate a change in trend: that emerging markets in the final period show more asymmetry than matured markets; an opposite of the first sub period. Thus in the Kenyan market which is an emerging one it might be expected to display high asymmetric volatility. The asymmetric volatility shows persistence in periods of great shocks and in generally low volatility markets. This asymmetry is related to short run more than long run volatility as observed by (Adrian & Rosenberg, 2008).

Causes of asymmetry in markets are deduced from their results and include: capital gains tax, short selling and derivatives trading. These and numerous other studies (Merton(1995), Holmes and Wong(2001)) have confirmed that countries with little capital gains tax, no short selling freedom but with derivatives trading present will have lower magnitudes of asymmetry. Regulation in Kenya does not allow short selling and derivatives have not yet been introduced but capital gains tax are present. The net impact of this on asymmetry as well as the relation of asymmetry on the volatility components are interesting areas for further research.

Aggregate market return is not the only component in explaining stock market volatility. The industry and firm level volatility also have a role. Firm level volatility referred to as idiosyncratic risk has been shown in (Campbell, Lettau, Malkiel, & Xu, 2001) to have the biggest role in explaining stock volatility: up to 70%. The increase in idiosyncratic risk over time has led to a decline in the explanatory power of market models that use the aggregate market as a factor. Despite increasing idiosyncratic risk, industry and market level volatility have remained stable over time

Increase in firm level volatility without subsequent increase in the market volatility is attributed to the declining correlations of stocks in the markets. Despite this observation, the paper is of the view that the number of stocks needed to be held in order to have a diversified portfolio has increased over time. This is contrary to what would be expected from declining correlations which ought to lead to the opposite conclusion¹², however, the authors explain it differently; they attribute the rise in number of assets needed in a diversified portfolio to be due to the increase in stock specific risk, which in essence counters the effect of declining correlations.

The trend in volatility is normally taken as a signal of the level of economic activity. The results of (Campbell, Lettau, Malkiel, & Xu, 2001) do not differ with this norm, they however show that the relation still holds for all the components of volatility: firm level, industry and market level.

As part of describing the different components of aggregate volatility, the authors use granger causality tests to suggest that Market volatility causes the other two components. This point raises a question in their whole work because: they claim that explanatory power of the market model has been declining due to rising idiosyncratic risk; but if the market return is the causal factor of the other two components, it should follow that the effect of increasing idiosyncratic risk should be captured in market models and thus not affect its explanatory power. This causality justifies the use of market volatility as a plausible return driving factor in this project.

Robustness of the results of their paper (Campbell, Lettau, Malkiel, & Xu, 2001) are tested by dampening the effect of the 1987 crash on the data and using weekly instead of daily data. The two modifications do not alter the conclusions of the authors. For their methodology the authors modify the CAPM to a form that would allow estimation of variances without the use of covariances or stock betas. First industry return is given a function of market return then stock return as a function of industry return.

¹² That fewer stocks would be needed for diversified portfolios.

Betas are eliminated by not using industry return directly but instead the weighted averages of the returns of industries in the market and weighted average of firm returns in each industry.

Though this method decomposes returns in the manner intended by the authors it is quite demanding in its implementation, for instance calculation of each industry's returns separately is too time consuming. These method might also give weak results in developing countries where listed stocks are too few to give an actual picture of the industry; whereas the authors worked with up to 2000 stocks listed in various exchanges in the US, an emerging economy like Kenya only has less than 100 companies listed.

Several causes are highlighted as the cause of idiosyncratic risk most of which (from simple observation) are not present in developing countries like Kenya. For instance, the rise in the number of early IPO's where long run profitability of the issuing companies is too uncertain thus increasing their firm specific risk. Though of less significance, increase in executive compensation using stock options has led to management engaging firms in riskier activities in order to seek higher return, this risk is transmitted to the stock price.

A possible explanation that might work in emerging markets highlighted by (Campbell, Lettau, Malkiel, & Xu, 2001) is the increase in uptake of debt by firms, increasing leverage increases the firms' riskiness and hence its idiosyncratic risk. Moving the thinking from causes to ways of reducing idiosyncratic risk, derivative trading intended to create more complete markets should lead to greater information content in prices about future cash flows of a firm thus reducing idiosyncratic risk confirmed by Grossman (1989).

With so much literature on the theory of asset pricing, some researchers delve into the testing of the models created. Methods of this are plenty but the most notable due to its wide use is the Fama Macbeth two pass regression (Macbeth & Fama, 1973) which has different ways of implementation. The various ways have been tested through simulation by (Shanken & Zhou, 2007).

They, (Shanken & Zhou, 2007), compare performance of the Ordinary Least Squares version of (Macbeth & Fama, 1973) two pass against the Weighted Least Squares, the Generalized Least Squares estimation, Generalized Method of Moments and the Maximum Likelihood Estimation. Some methods are more robust than others in accommodating for serial correlation and heteroskedasticity.

In brief, the Fama Macbeth procedure (Macbeth & Fama, 1973) for testing multifactor models simply put tests for the significance of the premiums derived from each of the factors. Given a Multifactor model for asset i over time

$$R_{it} = \beta_{i,0} + \beta_{i,1}F_{1,t} + \cdots + \beta_{i,n}F_{n,t} + e_{1,t}$$

The betas are estimated for many stocks over time. Then at each point in time, the returns on all stocks are regressed against the corresponding beta estimates. The resulting Factor loadings are taken as the premiums earned on each factor. For each beta variable the mean of the factor loadings are tested for significance using t tests.

The results of (Shanken & Zhou, 2007) show that the precision of the Maximum Likelihood estimator is close to or a bit lower than that of the GLS when measured using the root mean squared error. But both are more precise than the GLS or OLS in CAPM simulations. One interesting result is that even when a model is misspecified, standard errors become understated instead of overstated by 10%.

For testing the volatility component multifactor model in this project, the two pass regression in its GLS forms may be used but the number of stocks used for the tests will be reduced because of the computational intensity of this method and the lack of long time series for some stocks.

3 METHODOLOGY

3.1 Data Type and Sources

The daily NSE 20 Index returns from 2006 August are used in order to get a long data set that will capture the conditional and unconditional distribution of returns as suggested in (Chernov, Gallant, Ghysels, & Tauchen, 2003). The NSE 20 is preferred to the All Share index because it has a longer series and it can be a good proxy of the market portfolio, a result of the many sectors included in it.

Data on the Market returns and individual stocks, which will be of equal length will be obtained from Investing.com website. The risk-free rate will be the 3-month T-bill rate and this will be gotten from the Central Bank of Kenya website. The reason for using the T-bill rate is because of the wide literature that has employed it in the methodology section, also it has as many observations as the market data. Since the T-bill is given at monthly frequency, it is assumed to be constant for the the days of each month.

Stocks chosen to compare the two models will depend on the trading frequency and life span as a listed stock.

3.2 Research Design

The research design in this project is quantitative because data will be manipulated to extract information out of it. However, in a sense the research is also descriptive because in the end we will infer from our analysis whether short term and long term volatility can be used to better explain returns.

3.3 Model Setup

The method of decomposition of volatility into short run and long run components is based on the Engle and Lee Component GARCH which differs from the one used in (Adrian, T & Rosenberg, J 2008) which was:

$$R_{t+1}^M = \mu_t^M + \sqrt{v_t} \varepsilon_{t+1}$$

$$\ln \sqrt{v_t} = s_t + l_t$$

$$s_{t+1} = \theta_4 s_t + \theta_5 \varepsilon_{t+1} + \theta_6 \left(|\varepsilon_{t+1}| - \sqrt{\frac{2}{\pi}} \right)$$

$$l_{t+1} = \theta_7 + \theta_8 l_t + \theta_9 \varepsilon_{t+1} + \theta_{10} \left(|\varepsilon_{t+1}| - \sqrt{\frac{2}{\pi}} \right)$$

R_{t+1}^M is the market return in excess of the risk free rate

$\sqrt{v_t}$ is its conditional volatility

μ_t^M is the one period expected excess return

s_t and l_t are the short term and long term components respectively

The term $|\varepsilon_{t+1}| - \sqrt{\frac{2}{\pi}}$ is a shock to volatility and has a zero expected value given $\varepsilon_t \sim N(0,1)$. To justify the components; it has to be that $\theta_4 < \theta_8$ such that s_{t+1} reverts faster to its mean than the long run component.

Stationarity and non-negativity conditions on the variance of this approach were not given nor tested in the data analysis by the authors which is one criticism of this approach. This method can be estimated using Maximum Likelihood Estimation.

The component GARCH of Engle and Lee comes from the GARCH (1, 1) and is specified as:

$$R_t^M = E(R_t^M) + e_t$$

$$e_t = \sqrt{\sigma_t v_t}$$

$$\sigma_t^2 = l_t + \alpha_1 (e_{t-1}^2 - l_{t-1}) + \beta_1 (\sigma_{t-1}^2 - l_{t-1})$$

$$l_t = w + \phi (e_{t-1}^2 - \sigma_{t-1}^2) + p (l_{t-1} - w)$$

Where: $v_t \sim N(0,1)$

e_t is the mean corrected market return and,

σ_t^2 is the total variance/volatility

Other notations are the same as in the (Adrian & Rosenberg, 2008) approach.

For the variance to always be non-negative, the conditions imposed are: $\beta_1 > \phi$ and $\alpha_1 > 0$

The main criticism of this model as shown by (Cho & Elshahat, 2011) is that under specific conditions the long run variance component is indistinguishable from the total variance. This happens when β_1 and ϕ are small and/or α_1 is very close to zero. Despite that, the additive component GARCH model has been used by other researchers in research and working papers, for instance (Guo & Neely 2006). Also this model suffers from the same limitation as most GARCH models: that all shocks are weighted in the same way. The smooth transition and Regime Switching GARCH that can allow for this are severely difficult to estimate as highlighted by (Bauwens & Storti, 2007).

This component GARCH approach is open to modification, for instance adding other variables in the volatility equation if they are related to volatility. However, seeing that previous papers have not included other variables and possible data problems in those variables; for this project they are not included.

The estimation of this model can be implemented using the EViews Software and is much simpler to implement than the (Adrian & Rosenberg, 2008) approach.

What is similar about the two approaches is that current component estimates depend on previous (lagged) realizations of the components.

In addition to the previous two approaches, another possible way of decomposing volatility would be to use moving standard deviation- a concept similar to that of moving average. But in this method, it would be hard to justify what period is short enough to assume that the volatility experienced was short run and also what period length for long run volatility. In the component GARCH, the components are defined in terms of what they converge to. The short run component converges to zero while the long run component converges to a constant.

The previous approaches are for volatility decomposition into short term and long term components. Naturally, what should follow is the model for return estimation for individual stocks. For this I choose a linear form as is the norm in arbitrage pricing theory:

$$r_{it} = r_f + \beta_1(R_t^M - r_f) + \beta_2(\sigma_t^2 - l_t) + \beta_3 l_t + u_t$$

Where r_f is the risk free rate,

r_{it} is the stock return and

$E(R_M)$ is the expected market return estimated as the historical average.

The other variables are as specified in the Engle and Lee component GARCH approach. The performance of this model will be compared against that of the CAPM which has the form:

$$E(r_{it}) = r_f + \beta_4(R_t^M - r_f)$$

Both of the above models are fitted by regression in the EViews software.

3.4 Data Analysis

Before the model is implemented, the NSE20 series should be checked for stationarity using the Augmented Dickey Fuller¹³ unit root test. For a series y_t the following regression is carried out:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{s=1}^m a_s \Delta y_{t-s} + v_t$$

where Δy_t is the change lag difference of y_t and

βt is the trend component

The null and alternative hypothesis are: $H_0: \gamma = 0$ (nonstationary and unit root present) and $H_a: \gamma < 0$

A *tau* statistic is used for the ADF test. The critical values are provided in the statistical package used.

The decision to include the trend component and the constant is dependent on visual evidence of whether the series is slow turning around zero or a non-zero value, or around a trend line. Nevertheless, the hypotheses remain the same regardless of presence of the two terms.

¹³ Referred to as ADF test going forward.

The number of lags chosen will depend on the Akaike information criterion and the Bayesian criterion which should be low for lags to be acceptable. If stationarity is found present in the levels form of the series, then it will be differenced and the new series is again tested for stationarity.

However the data may have visual evidence of regimes (i.e. separate periods where the characteristics of the data change and the changes persist). If this happens, then a different stationarity test is needed. The Phillips-Peron test will be useful in the presence of said structural breaks. It involves a regression of the series y_t on its previous lag y_{t-1} with or without the time trend then converting the *tau* statistic to a modified Z statistic which under the null hypothesis follows the Dickey-Fuller distribution. Rejection of stationarity or failure thereof is easily verified from the EViews software output using p-values.

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3.5 Post Estimation

To compare the performance of this component model against the standard CAPM, the adjusted R^2 is used.

4 DATA ANALYSIS

The analysis has been done on both daily and weekly data to enhance robustness of results and inference. The section following is thus divided under those two headings.

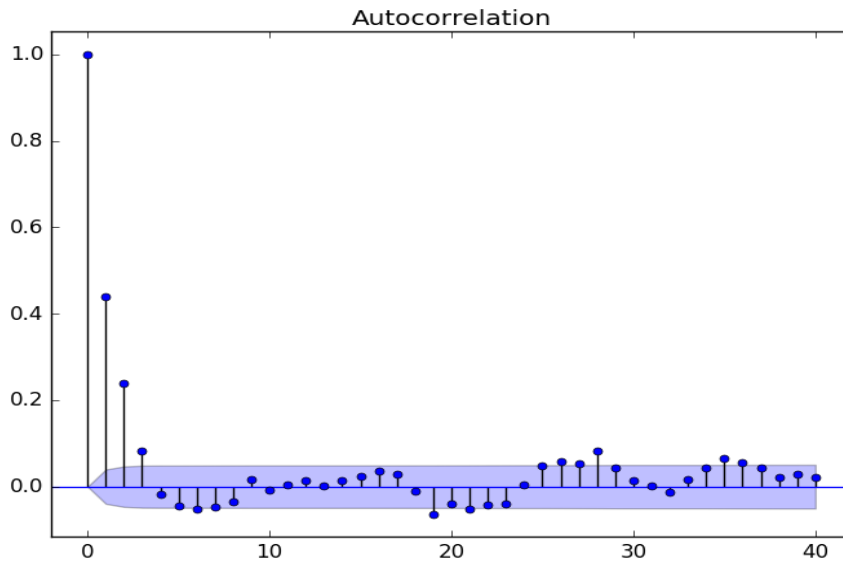
4.1 Daily analysis

The analysis begins by finding a mean equation for the daily market return using ARMA models because of the lack of other factors affect the market and have an observed daily frequency. Using (partial)correlogram plots, the most likely ARMA model seems to have a maximum of 3 MA or AR terms. Upon trial and error of the possible combinations, ARMA (2,2) is selected based on Akaike Information Criterion.

Table 1: Akaike Criterion for various ARMA orders

AR / MA	0	1	2	3	4
0	-6.6494	-6.8156	-6.8592	-6.8708	-6.8701
1	-6.8643	-6.8659	-6.8686	-6.8697	-6.8689
2	-6.8661	-6.8662	-6.8709	-6.8701	-6.8696
3	-6.8675	-6.8705	-6.8700	-6.8692	-6.8688
4	-6.8700	-6.8701	-6.8702	-6.8692	-6.8692

Figure 1: Correlogram of realized daily market returns



Inference for the selected ARMA model is valid because the unit tests conducted agree that the series is stationary. In the ADF test that was described earlier, there was no need to include either the intercept or trend term as the plot of the daily return was quite noisy. Only one lag was chosen based on the Akaike criterion. The plot of return showed no structural breaks therefore the Phillips-

Peron test was not necessary. The component GARCH is then estimated via maximum likelihood and the results displayed in the following table:

Table 2: Results from CGARCH fit of daily returns.

Coefficient	Estimate	Std. Error	z-Statistic	Prob.
w	0.0000545	0.00000619	8.809967	0.0000
p	0.982919	0.003774	260.4684	0.0000
ϕ	0.046817	0.009336	5.014486	0.0000
α_1	0.235980	0.020572	11.47109	0.0000
β_1	0.449650	0.053607	8.387836	0.0000
R^2	0.188421			
Adjusted R^2	0.187419			
S.E. of regression	0.007847			
Sum squared residuals	0.149559			
Log likelihood	8746.608			

All the coefficients of the variance equation are significant as supported by the 0 p-values. The R^2 of the model is also a double-digit percent which provides more confidence in the model. The power of convergence of the long run component l_t to w is within its typical range of 0.99 ($p = 0.982$). Therefore, the long run volatility of daily market return converges slowly.

Estimating the CAPM using daily data leads to an average R^2 of 63.62% in the 26 stocks. The market premium is a significant factor in all 26 regressions whereas the significance of the alpha is varying among the stocks. The results for this are in [Table 6](#) and [Table 7](#).

Once the volatility components are included, the market premium remains jointly significant in all 26 models. However, the long run variance of the market is jointly insignificant in all models with a p-value of 98%. When both components are tested jointly for significance in all models they turn out to be jointly insignificant with a p value of 59%

4.2 Weekly analysis

Similar to the daily analysis we start by finding a suitable mean equation from the weekly market return. Since both the partial correlogram and correlograms were unclear, no apparent number of MA or AR terms could be inferred. The only method left to choose the model terms was to use the software to search through the possible AR and MA terms and select the best model using Akaike Information criteria. Two seasonal ARMA terms at period 10 each seemed to improve the AIC of the model hence were included. The results are shown in the following table:

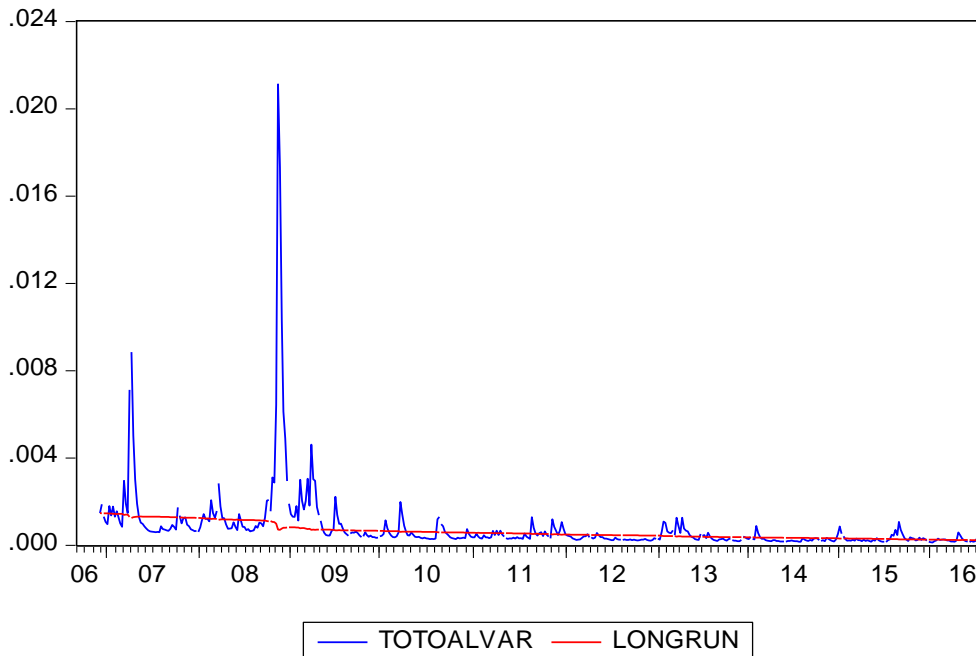
Table 3: Results for ARMA fit of weekly returns

Coefficient	Estimate	Std. Error	t-Statistic	Prob.
C	-0.000888	0.001445	-0.614073	0.5395
AR(1)	-0.879591	0.035280	-24.93175	0.0000
AR(2)	-0.877297	0.035389	-24.79009	0.0000
SAR(10)	-0.418194	0.130804	-3.197106	0.0015
MA(1)	0.874271	0.025019	34.94472	0.0000
MA(2)	0.949120	0.024713	38.40611	0.0000
SMA(10)	0.524613	0.126623	4.143128	0.0000
R-squared	0.112703			
Adjusted R-squared	0.101028			
S.E. of regression	0.028339			
Sum squared resid	0.366204			
Log likelihood	996.4718			
F-statistic	9.653348			

For the weekly market return mean equation, R^2 , AIC and BIC are all weaker than from the daily analysis. The mean equation itself has an insignificant constant (p-value of 53%) but nevertheless it used for the CGARCH estimation.

The component GARCH is estimated using the above mean equation and the estimated total variance and long run component are graphed below:

Figure 2: Graph of long run and total variance over the sample period (2006 -2016)



From the Figure 2 it seems that the volatility estimates are not significantly different from zero because they are all below 0.025. However, when compared to the total sample variance of the whole return series which was 0.0009, it turns out that the model is not so far off.

The coefficients from the GARCH model fit are in the following table. The model is not such a close fit because two of the coefficients are insignificant. Notwithstanding, the analysis continues to determine if the components from such a loosely fitting model will be significant as pricing factors.

Table 4: CGARCH results of weekly returns

Coefficient	Estimate	Std. Error	z-Statistic	Prob.
w	-0.000116	0.001146	-0.101556	0.9191
p	0.997301	0.005057	197.2159	0.0000
ϕ	-0.004474	0.007038	-0.635700	0.5250
α_1	0.260869	0.054686	4.770271	0.0000
β_1	0.531948	0.081449	6.531090	0.0000
R^2	-0.040380			
Adjusted R^2	-0.055975			
S.E. of regression	0.030788			
Sum squared residuals	0.442682			
Log likelihood	1114.930			
Durbin-Watson stat	2.289203			

The constant that the long run component converges to in this model, w seems to be insignificant based on the high p-value. The sample weekly returns supports this because it had a variance that was very close to zero for the entire sample period.

Firstly, the CAPM regression from 26 stocks are reported. The R^2 for the different stocks varied greatly from 22% up to 65%. The table for all R^2 is included in the appendix. The average R^2 was 46.53%.

The weekly market premium as the only factor in CAPM turned out to be jointly significant in all 26 regressions and at all levels of significance. This is supported by a p value of 0.0000 and an F-statistic of 562.23.

Now to compare the performance of the CAPM with a multifactor model that includes the components, the 26 regressions are repeated with components included. The average R^2 increases to 46.93% which is a small increase but probably it would be more significant if a stronger mean equation had been used for estimating the GARCH.

The joint significance of all the parameters in the 26 models is robust however the significance of longrun variance is rejected at all significance levels; its p-value is 38%. The short run component is however significant at the 10% level.

4.3 Cross sectional significance

Using the coefficient estimates from the time series returns regressions, I estimate the fama Macbeth 2 step regression parameters to determine premiums for each of my factors. The estimation is done only for weekly series because daily returns have too much noise in beta estimates.

Table 5: Fama Macbeth procedure results of weekly returns

	Unexplained return	Market return premium	Long run variance premium	Short run variance premium
Mean	0.001591	-0.006180	-0.0000835	-0.000480
P-value	0.000000	0.000000	0.000000	0.000000
t-statistic	1.053952	-1.337590	-0.906387	-1.959067
Std. Dev	0.033313	0.101960	0.002033	0.005407

As displayed in Table 5, both longrun and short run components are significant in explaining cross sectional variation of returns because all p values are 0%. This contrasts with the time series results that disapproved of the components explanatory power.

The significance of the unexplained return in the table suggests that there are some additional factors that are not captured which contribute to variation in cross sectional return. This return is however very small, less than 1%.

The volatility components have negative signs which suggests that taking on extra risk is not rewarded but rather lowers expected return. This is contrary to intuition but the negative price of risk factors has been found numerous times in past works as referenced in the literature review. Perhaps the reason for the negative sign is due to the measurement error in the coefficient estimates of time series regression. Using of portfolios to carry out the analysis as is common in literature might have reduced this error. However, previous literature in advanced markets had more than 1,000 stocks to create robust portfolios. I find that 26 stocks are too few to reduce measurement error even when portfolios are formed.

5 CONCLUSION

From both the weekly and daily analysis it is clear that volatility components are not significant pricing factors in the Kenyan market at least for the selected stocks. This might stem from the lack of a strong mean equation of the market return. If more variables had been used to estimate the GARCH model perhaps the volatility components would have been more significant in explaining return variation over time.

The surprising result is that in this market the CAPM model is stronger when using noisy daily data than when using weekly data. This is evidenced by the R^2 from the analysis. Perhaps when an analysis based on portfolios is used and more noise is cancelled out, the conclusion might change.

Despite its poor performance as an explanatory variable, the component GARCH seems to capture the volatility trend of the market pretty well. Case in point is the post election violence period of 2008. While long run volatility was still quite stable at that time, short run volatility shows spikes that could have been used by an investor to detect the increased riskiness of the market.

In the reviewed literature, volatility was shown to be related to the business cycle and if this is true then one might assume that the business cycle of the Kenyan economy does not influence the pricing of stocks. This would however be inappropriate because the influence of economic factors on stock prices is a given. The conclusion should be that volatility components are not a good proxy for the business cycle hence show no influence on pricing.

Despite the time series rejection of the volatility components, the Fama Macbeth procedure concludes that the volatility components have a negative premium for returns. This is important to investors because it means that portfolios will have to be constructed to minimize influence of the volatility components.

6 Tables

Asset	CAPM WEEKLY		CAPM DAILY	
	R ²	F statistic	R ²	F statistic
sasini	0.3563	261.8692	0.5996	3640.829
sameer	0.3502	254.8794	0.5178	2609.995
cfc	0.2033	120.7078	0.8203	11094
barclays	0.6225	779.8492	0.507	2500.314
dtb	0.5856	668.4033	0.7144	6081.431
equity	0.3628	269.2764	0.5406	2860.383
housingfinance	0.488	450.7423	0.6708	4953.714
kcb	0.6533	891.4865	0.8221	11236.85
nbk	0.5072	486.7796	0.6388	4300.272
nic	0.3703	278.1855	0.5878	3466.343
stanchart	0.5913	684.4198	0.8196	11044.71
kq	0.4392	370.4408	0.7172	6164.607
nmg	0.5879	674.8813	0.7589	7652.061
standardme~a	0.2284	140.0003	0.4824	2265.41
tpseastern~a	0.4665	413.5481	0.6406	4333.113
scangroup	0.4624	406.9107	0.6301	4140.786
bamburicem~t	0.4862	447.5023	0.7589	7650.724
eacables	0.5267	526.3461	0.664	4803.1
kengen	0.5097	491.7204	0.7403	6928.281
kenyapower	0.6474	868.5224	0.0626	162.392
total	0.4385	369.3782	0.6589	4696.192
jubileehol~s	0.4411	373.3639	0.6446	4409.765
centum	0.5972	701.2743	0.684	5260.936
bat	0.3848	295.8268	0.7011	5700.926
ungagroup	0.328	230.8984	0.5237	2672.786
AVERAGE	0.465392		0.636244	

Table 6: CAPM Performance

Equation	COMPONENTS WEEKLY		COMPONENTS DAILY	
	R ²	F statistic	R ²	F
sasini	0.3599	88.26396	0.5996	3640.829
sameer	0.354	86.034	0.5178	2609.995
cfc	0.2149	42.98221	0.8203	11094
barclays	0.6225	258.899	0.507	2500.314
dtb	0.5863	222.4597	0.7144	6081.431
equity	0.3802	96.3077	0.5406	2860.383
housingfin~e	0.4901	150.9144	0.6708	4953.714
kcb	0.6569	300.6163	0.8221	11236.85
nbk	0.5086	162.5003	0.6388	4300.272
nic	0.3715	92.81429	0.5878	3466.343
stanchart	0.5919	227.6721	0.8196	11044.71
kq	0.4423	124.5139	0.7172	6164.607
nmg	0.5966	232.2018	0.7589	7652.061
standardme~a	0.2312	47.20174	0.4824	2265.41
tpseastern~a	0.4684	138.3154	0.6406	4333.113
scangroup	0.4658	136.9021	0.6301	4140.786
bamburicem~t	0.49	150.8188	0.7589	7650.724
eacables	0.5278	175.4615	0.664	4803.1
kengen	0.5117	164.5267	0.7403	6928.281
kenyapower	0.6478	288.7194	0.0626	162.392
total	0.4455	126.1229	0.6589	4696.192
jubileehol~s	0.4513	129.1404	0.6446	4409.765
centum	0.6039	239.3226	0.684	5260.936
bat	0.3856	98.52675	0.7011	5700.926
ungagroup	0.3284	76.76634	0.5237	2672.786
AVERAGE	46.9%		63.62%	

Table 7: Components performance

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